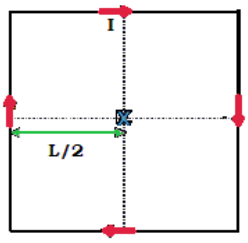
LEI DE AMPERE E O CAMPO MAGNÉTICO
AMPERE'S LAW AND THE MAGNETIC FIELD

1) Considere uma espira quadrada de lado **L** que se encontra no vazio e que é percorrida por uma corrente **I**. (Escolha o referencial que considerar adequado.) Assume a square loop with side **L** that is in the vacuum is crossed by a current **I**. (Select an adequate frame.)



a) Encontre a expressão do campo magnético, **H**, no centro da espira. Find the expression of the magnetic field, **H**, at the center of the loop.

$$\vec{H} = \oint \frac{I d\vec{l}}{4\pi R^2} \times \vec{a}_R$$

Resolução Resolution

$$\vec{H} = \oint \frac{I d\vec{l}}{4\pi R^2} \times \vec{a}_R = \vec{H}_1 + \vec{H}_2 + \vec{H}_3 + \vec{H}_4$$

$$\vec{H}_1 = \int_{-L/2}^{L/2} \frac{I dx \vec{a}_x}{4\pi R^2} \times \vec{a}_{R1}$$

$$\vec{H}_2 = \int_{L/2}^{-L/2} \frac{I dy \vec{a}_y}{4\pi R^2} \times \vec{a}_{R2}$$

$$\vec{H}_3 = \int_{L/2}^{-L/2} \frac{I dx \vec{a}_x}{4\pi R^2} \times \vec{a}_{R3}$$

$$\vec{H}_4 = \int_{-L/2}^{L/2} \frac{I dy \vec{a}_y}{4\pi R^2} \times \vec{a}_{R4}$$

$$\vec{H}_1 = \int_{-L/2}^{L/2} \frac{I dx \vec{a}_x}{4\pi (x^2 + L^2/4)} \times \frac{(0\vec{a}_x + 0\vec{a}_y) - (x\vec{a}_x + \frac{L}{2}\vec{a}_y)}{\sqrt{x^2 + L^2/4}}$$

$$\vec{H}_2 = \int_{L/2}^{-L/2} \frac{I dy \vec{a}_y}{4\pi (L^2/4 + y^2)} \times \frac{-\frac{L}{2}\vec{a}_x - y\vec{a}_y}{\sqrt{L^2/4 + y^2}}$$

$$\vec{H}_3 = \int_{L/2}^{-L/2} \frac{I dx \vec{a}_x}{4\pi (x^2 + L^2/4)} \times \frac{-x\vec{a}_x + \frac{L}{2}\vec{a}_y}{\sqrt{x^2 + L^2/4}}$$

$$\vec{H}_4 = \int_{-L/2}^{L/2} \frac{I dy \vec{a}_y}{4\pi (L^2/4 + y^2)} \times \frac{\frac{L}{2}\vec{a}_x - y\vec{a}_y}{\sqrt{L^2/4 + y^2}}$$

$$\vec{H}_1 = \int_{-L/2}^{L/2} \frac{I dx \vec{a}_x}{4\pi(x^2+L^2/4)^{3/2}} \times (-x\vec{a}_x - \frac{L}{2}\vec{a}_y)$$

$$\vec{H}_2 = \int_{L/2}^{-L/2} \frac{I dy \vec{a}_y}{4\pi(L^2/4+y^2)^{3/2}} \times (-\frac{L}{2}\vec{a}_x - y\vec{a}_y)$$

$$\vec{H}_3 = \int_{L/2}^{-L/2} \frac{I dx \vec{a}_x}{4\pi(x^2+L^2/4)^{3/2}} \times (-x\vec{a}_x + \frac{L}{2}\vec{a}_y)$$

$$\vec{H}_4 = \int_{-L/2}^{L/2} \frac{I dy \vec{a}_y}{4\pi(L^2/4+y^2)^{3/2}} \times (\frac{L}{2}\vec{a}_x - y\vec{a}_y)$$

$$\vec{H}_1 = -\frac{IL}{8\pi}\vec{a}_z \int_{-L/2}^{L/2} \frac{dx}{(x^2+L^2/4)^{3/2}}$$

$$\vec{H}_2 = -\frac{IL}{8\pi}\vec{a}_z \int_{-L/2}^{L/2} \frac{dy}{(L^2/4+y^2)^{3/2}}$$

$$\vec{H}_3 = -\frac{IL}{8\pi}\vec{a}_z \int_{-L/2}^{L/2} \frac{dx}{(x^2+L^2/4)^{3/2}}$$

$$\vec{H}_4 = -\frac{IL}{8\pi}\vec{a}_z \int_{-L/2}^{L/2} \frac{dy}{(L^2/4+y^2)^{3/2}}$$

$$\vec{H}_1 = -\frac{IL}{8\pi}\vec{a}_z\, 2\frac{8L/2}{L^2\sqrt{L^2+4(L/2)^2}}$$

$$\vec{H}_2 = -\frac{IL}{8\pi}\vec{a}_z\, 2\frac{8L/2}{L^2\sqrt{L^2+4(L/2)^2}}$$

$$\vec{H}_3 = -\frac{IL}{8\pi}\vec{a}_z\, 2\frac{8L/2}{L^2\sqrt{L^2+4(L/2)^2}}$$

$$\vec{H}_4 = -\frac{IL}{8\pi}\vec{a}_z\, 2\frac{8L/2}{L^2\sqrt{L^2+4(L/2)^2}}$$

$$\vec{H}_1 = \frac{IL}{8\pi}\vec{a}_z \int_{-L/2}^{L/2} \frac{-dx}{(x^2+L^2/4)^{3/2}}$$

$$\vec{H}_2 = \frac{IL}{8\pi}\vec{a}_z \int_{L/2}^{-L/2} \frac{dy}{(L^2/4+y^2)^{3/2}}$$

$$\vec{H}_3 = \frac{IL}{8\pi}\vec{a}_z \int_{L/2}^{-L/2} \frac{dx}{(x^2+L^2/4)^{3/2}}$$

$$\vec{H}_4 = \frac{IL}{8\pi}\vec{a}_z \int_{-L/2}^{L/2} \frac{-dy}{(L^2/4+y^2)^{3/2}}$$

$$\vec{H}_1 = -\frac{IL}{8\pi}\vec{a}_z \left[\frac{8x}{L^2\sqrt{L^2+4x^2}}\right]_{-L/2}^{L/2}$$

$$\vec{H}_2 = -\frac{IL}{8\pi}\vec{a}_z \left[\frac{8y}{L^2\sqrt{L^2+4y^2}}\right]_{-L/2}^{L/2}$$

$$\vec{H}_3 = -\frac{IL}{8\pi}\vec{a}_z \left[\frac{8x}{L^2\sqrt{L^2+4x^2}}\right]_{-L/2}^{L/2}$$

$$\vec{H}_4 = -\frac{IL}{8\pi}\vec{a}_z \left[\frac{8y}{L^2\sqrt{L^2+4y^2}}\right]_{-L/2}^{L/2}$$

$$\vec{H}_1 = -\frac{I}{\pi L\sqrt{2}}\vec{a}_z$$

$$\vec{H}_2 = -\frac{I}{\pi L\sqrt{2}}\vec{a}_z$$

$$\vec{H}_3 = -\frac{I}{\pi L\sqrt{2}}\vec{a}_z$$

$$\vec{H}_4 = -\frac{I}{\pi L\sqrt{2}}\vec{a}_z$$

$$\vec{H} = \vec{H}_1 + \vec{H}_2 + \vec{H}_3 + \vec{H}_4 = -\frac{4I}{\pi L\sqrt{2}}\vec{a}_z = -\frac{2I\sqrt{2}}{\pi L}\vec{a}_z$$

$$\vec{H} = -\frac{2I\sqrt{2}}{\pi L}\vec{a}_z \qquad \text{(A/m)}$$

Resposta Answer

$$\vec{H} = -\frac{2I\sqrt{2}}{\pi L}\vec{a}_z \qquad \text{(A/m)}$$

b) Encontre a expressão da densidade de fluxo magnético, **B**, no centro da espira. Find the expression of the magnetic flux density, **B**, in the loop center.

$$\vec{B} = \mu_o \mu_r \vec{H}$$

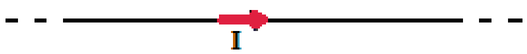
Resolução Resolution

$$\vec{B} = \mu_o \mu_r \left(-\frac{2I\sqrt{2}}{\pi L}\vec{a}_z \right) \qquad \vec{B} = -\mu_o \mu_r \frac{2I\sqrt{2}}{\pi L}\vec{a}_z \quad \text{(T)}$$

Resposta Answer

$$\vec{B} = -\mu_o \mu_r \frac{2I\sqrt{2}}{\pi L}\vec{a}_z \qquad \text{(T)}$$

2) Considere um condutor rectilíneo infinito de raio ***a*** que se encontra no vazio e é percorrido por uma corrente constante de intensidade **I**. (Escolha o referencial que considerar adequado.) Assume an infinite straight conductor of radius **a** that lies in the vacuum. The intensity of current, **I**, through conductor is constant. (Choose one appropriate frame.)



$$\vec{H} = \oint \frac{I d\vec{l}}{4\pi R^2} \times \vec{a}_R \qquad \vec{B} = \mu_o \mu_r \vec{H}$$

a) Encontre a expressão do campo magnético, **H**, gerado pela corrente que atravessa o condutor à distância de 10 Cm do condutor. Find the expression of the magnetic field, **H,** generated by the current through the conductor at a distance of 10 Cm from it.

$$\vec{H} = \oint \frac{I d\vec{l}}{4\pi R^2} \times \vec{a}_R$$

Escolhendo um referencial cartesiano com z ao longo do condutor.

$$\vec{H} = \lim_{K\to\infty} \frac{I}{4\pi} \int_{-K}^{K} \frac{dz\,\vec{a}_z}{0.1^2 + z^2} \times \frac{0.1\vec{a}_x - z\vec{a}_z}{\sqrt{0.1^2 + z^2}}$$

$$\vec{H} = \lim_{K\to\infty} \frac{I}{4\pi} \int_{-K}^{K} \frac{0.1 dz\,\vec{a}_y}{(0.1^2 + z^2)^{3/2}}$$

$$\vec{H} = 5\frac{I}{2\pi}\vec{a}_y \lim_{K\to\infty}\left[\frac{z}{\sqrt{z^2 + 1/100}}\right]_{-K}^{K}$$

$$\vec{H} = 5\frac{I}{\pi}\vec{a}_y 1$$

$$\vec{H} = \frac{5I}{\pi}\vec{a}_y$$

Resposta Answer

$$\vec{H} = \frac{5I}{\pi}\vec{a}_y$$

b) Encontre a expressão geral da magnitude da densidade de fluxo magnético, **B**, gerada pela corrente que atravessa o condutor à distância **R** do condutor. Find the expression of the magnetic flux density magnitude, **B**, generated by the current through the conductor at a distance **R** from the conductor.

$$\vec{H} = \oint \frac{I d\vec{l}}{4\pi R^2} \times \vec{a}_R$$

Resolução Resolution

Escolhendo um referencial cartesiano com z ao longo do condutor. Tendo em conta a simetria do campo em torno do condutor o ponto a considerar pode ser o (R, 0, 0);

$$\vec{H} = \lim_{K\to\infty} \frac{I}{4\pi} \int_{-K}^{K} \frac{dz\vec{a}_z}{R^2 + z^2} \times \frac{R\vec{a}_x - z\vec{a}_z}{\sqrt{R^2 + z^2}}$$

$$\vec{H} = \lim_{K\to\infty} \frac{I}{4\pi} \int_{-K}^{K} \frac{R dz\vec{a}_y}{(R^2 + z^2)^{3/2}}$$

$$\vec{H}=\frac{I}{4\pi}\vec{a}_y \lim_{K\to\infty}\left[\frac{z}{R\sqrt{z^2+R^2}}\right]_{-K}^{K}$$

$$\vec{H}=\frac{I}{4\pi R}\vec{a}_y\, 2\lim_{K\to\infty}\left(\frac{z}{\sqrt{z^2+R^2}}\right)$$

$\vec{H}=\frac{I}{2\pi R}\vec{a}_y$ como $\vec{B}=\mu_o\mu_r\vec{H}$ tem-se $\vec{B}=\mu_o\mu_r\frac{I}{2\pi R}\vec{a}_y$

Para o modulo de B vem: $B=\mu_o\mu_r\frac{I}{2\pi R}$

Resposta Answer

$$B=\mu_o\mu_r\frac{I}{2\pi R}$$

3) Considere um campo magnético cuja função vector potencial magnético no referencial cartesiano é o apresentado na equação ao lado. Assume a magnetic field whose magnetic vector potential function in the Cartesian reference frame is the presented in equation at left.

$$\vec{A}=y\cos(2x)\vec{a}_x+(3y+e^x)\vec{a}_z$$

a) Encontre a expressão do campo magnético, **H**, e da densidade de fluxo magnético, **B**. Find the expression of the magnetic field, **H**, and the magnetic flux density, **B**.

$\vec{B}=\nabla\times\vec{A}$

$$\nabla\times\vec{A}=\left(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right)\vec{a}_x+\left(\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}\right)\vec{a}_y+\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)\vec{a}_z$$

Resolução Resolution

$$\vec{B}=\nabla\times\vec{A}=\left(\frac{\partial(3y+e^x)}{\partial y}-\frac{\partial 0}{\partial z}\right)\vec{a}_x+\left(\frac{\partial y\cos(2x)}{\partial z}-\frac{\partial(3y+e^x)}{\partial x}\right)\vec{a}_y+\left(\frac{\partial 0}{\partial x}-\frac{\partial y\cos(2x)}{\partial y}\right)\vec{a}_z$$

$$\vec{B}=(3-0)\vec{a}_x+\left(0-e^x\right)\vec{a}_y+(0-\cos(2x))\vec{a}_z=3\vec{a}_x-e^x\vec{a}_y-\cos(2x)\vec{a}_z$$

$\vec{B}=3\vec{a}_x-e^x\vec{a}_y-\cos(2x)\vec{a}_z$ como $\vec{B}=\mu_o\mu_r\vec{H}$ vem : $\vec{H}=\frac{3\vec{a}_x-e^x\vec{a}_y-\cos(2x)\vec{a}_z}{\mu_o\mu_r}$

Resposta Answer

$$\vec{H}=\frac{3\vec{a}_x-e^x\vec{a}_y-\cos(2x)\vec{a}_z}{\mu_o\mu_r} \qquad \vec{B}=3\vec{a}_x-e^x\vec{a}_y-\cos(2x)\vec{a}_z$$

b) Encontre **H** e **B** no ponto (1, 0, 3). Find H and, **B,** at the point (1,0,3).

$$\vec{\mathrm{H}}=\frac{3\vec{\mathrm{a}}_{\mathrm{x}}-e^{x}\,\vec{\mathrm{a}}_{\mathrm{y}}-\cos(2x)\,\vec{\mathrm{a}}_{\mathrm{z}}}{\mu_o\mu_r}\qquad\qquad \vec{\mathrm{B}}=3\vec{\mathrm{a}}_{\mathrm{x}}-e^{x}\,\vec{\mathrm{a}}_{\mathrm{y}}-\cos(2x)\,\vec{\mathrm{a}}_{\mathrm{z}}$$

Resolução Resolution

$$\vec{\mathrm{H}}=\frac{3\vec{\mathrm{a}}_{\mathrm{x}}-e^{1}\,\vec{\mathrm{a}}_{\mathrm{y}}-\cos(2\times 1)\,\vec{\mathrm{a}}_{\mathrm{z}}}{\mu_o\mu_r}\qquad\qquad \vec{\mathrm{H}}=\frac{3\vec{\mathrm{a}}_{\mathrm{x}}-e\,\vec{\mathrm{a}}_{\mathrm{y}}-\cos(2)\,\vec{\mathrm{a}}_{\mathrm{z}}}{\mu_o\mu_r}$$

$$\vec{\mathrm{B}}=3\vec{\mathrm{a}}_{\mathrm{x}}-e^{1}\,\vec{\mathrm{a}}_{\mathrm{y}}-\cos(2\times 1)\,\vec{\mathrm{a}}_{\mathrm{z}}\qquad\qquad \vec{\mathrm{B}}=3\vec{\mathrm{a}}_{\mathrm{x}}-e\,\vec{\mathrm{a}}_{\mathrm{y}}-\cos(2)\,\vec{\mathrm{a}}_{\mathrm{z}}$$

Resposta Answer

$$\vec{\mathrm{H}}=\frac{3\vec{\mathrm{a}}_{\mathrm{x}}-e\,\vec{\mathrm{a}}_{\mathrm{y}}-\cos(2)\,\vec{\mathrm{a}}_{\mathrm{z}}}{\mu_o\mu_r}\qquad\qquad \vec{\mathrm{B}}=3\vec{\mathrm{a}}_{\mathrm{x}}-e\,\vec{\mathrm{a}}_{\mathrm{y}}-\cos(2)\,\vec{\mathrm{a}}_{\mathrm{z}}$$

c) Encontre a expressão da densidade de corrente **J** responsável pelo campo. Find the expression of the current density, **J,** related to the field.

$$\nabla^2\vec{\mathrm{A}}=-\mu\,\vec{\mathrm{J}}\qquad\qquad \nabla^2\vec{\mathrm{A}}=\nabla^2\mathrm{A}_x\,\vec{a}_x+\nabla^2\mathrm{A}_y\,\vec{a}_y+\nabla^2\mathrm{A}_z\,\vec{a}_z$$

$$\vec{\mathrm{A}}=y\cos(2x)\vec{a}_x+(3y+e^{x})\vec{a}_z\qquad\qquad lap(\mathrm{A})=\nabla^2\mathrm{A}=\frac{\partial^2\mathrm{A}}{\partial\mathrm{x}^2}+\frac{\partial^2\mathrm{A}}{\partial\mathrm{y}^2}+\frac{\partial^2\mathrm{A}}{\partial\mathrm{z}^2}$$

Resolução Resolution

$$-\mu\,\vec{\mathrm{J}}=\nabla^2\mathrm{A}_x\,\vec{a}_x+\nabla^2\mathrm{A}_y\,\vec{a}_y+\nabla^2\mathrm{A}_z\,\vec{a}_z$$

$$-\mu\,\vec{\mathrm{J}}=\left(\frac{\partial^2\mathrm{A}_x}{\partial\mathrm{x}^2}+\frac{\partial^2\mathrm{A}_x}{\partial\mathrm{y}^2}+\frac{\partial^2\mathrm{A}_x}{\partial\mathrm{z}^2}\right)\vec{a}_x+\left(\frac{\partial^2\mathrm{A}_y}{\partial\mathrm{x}^2}+\frac{\partial^2\mathrm{A}_y}{\partial\mathrm{y}^2}+\frac{\partial^2\mathrm{A}_y}{\partial\mathrm{z}^2}\right)\vec{a}_y+\left(\frac{\partial^2\mathrm{A}_z}{\partial\mathrm{x}^2}+\frac{\partial^2\mathrm{A}_z}{\partial\mathrm{y}^2}+\frac{\partial^2\mathrm{A}_z}{\partial\mathrm{z}^2}\right)\vec{a}_z$$

$$\begin{aligned}-\mu\,\vec{\mathrm{J}}=&\left(\frac{\partial^2 y\cos(2x)}{\partial\mathrm{x}^2}+\frac{\partial^2 y\cos(2x)}{\partial\mathrm{y}^2}+\frac{\partial^2 y\cos(2x)}{\partial\mathrm{z}^2}\right)\vec{a}_x+\\&\left(\frac{\partial^2 0}{\partial\mathrm{x}^2}+\frac{\partial^2 0}{\partial\mathrm{y}^2}+\frac{\partial^2 0}{\partial\mathrm{z}^2}\right)\vec{a}_y+\\&\left(\frac{\partial^2(3y+e^{x})}{\partial\mathrm{x}^2}+\frac{\partial^2(3y+e^{x})}{\partial\mathrm{y}^2}+\frac{\partial^2(3y+e^{x})}{\partial\mathrm{z}^2}\right)\vec{a}_z\end{aligned}$$

$$-\mu\vec{J} = \left(-4y\cos(2x) + 0 + 0\right)\vec{a}_x + \left(0 + 0 + 0\right)\vec{a}_y + \left(e^x + 0 + 0\right)\vec{a}_z$$

$$-\mu\vec{J} = -4y\cos(2x)\vec{a}_x + e^x\vec{a}_z \qquad \vec{J} = \frac{4y\cos(2x)\vec{a}_x - e^x\vec{a}_z}{\mu}$$

Resposta Answer

$$\vec{J} = \frac{4y\cos(2x)\vec{a}_x - e^x\vec{a}_z}{\mu}$$

4) Considere uma região cilíndrica do espaço com raio **a** que se encontra no vazio e é percorrida por uma corrente de densidade **J**. Encontre a expressão do campo magnético, **H**, e da densidade de fluxo magnético, **B**, gerados pelo condutor. (Escolha o referencial que considerar adequado.) Assume an cylindrical region with radius **a** which is in the vacuum. It is crossed by a current density **J.** Find the expression of the magnetic field, **H**, and of the magnetic flux density, **B**, generated by the density of current **J**.(Choose one appropriate frame.)

$$I = J\pi a^2 \qquad H = \frac{I}{2\pi R}$$

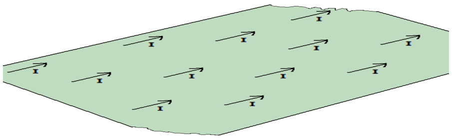
Resolução Resolution

$$H = \frac{J\pi a^2}{2\pi R} \qquad H = \frac{J a^2}{2R} \qquad \text{como} \quad \vec{B} = \mu_o \vec{H} \qquad \text{vem} \quad B = \mu_o \frac{J a^2}{2R}$$

Resposta Answer

$$H = \frac{J a^2}{2R} \qquad B = \mu_o \frac{J a^2}{2R}$$

5) Considere uma película plana infinita, que se encontra no vazio percorrida por uma corrente de densidade **K**. Encontre a expressão do campo magnético, **H**, e da densidade de fluxo magnético, **B**, gerados pela película. (Escolha o referencial que considerar adequado.) Assume an infinite flat film in vacuum where flows a current density **K**. Find the expression of the magnetic field **H** and the magnetic flux density, **B**, generated by the film. (Choose one appropriate frame.)



$$\vec{H} = \oint \frac{I d\vec{l}}{4\pi R^2} \times \vec{a}_R$$

Resolução Resolution

$\vec{H} = \oint \frac{I d\vec{l}}{4\pi R^2} \times \vec{a}_R$ para uma porção infinitesimal da largura da fita condutora a corrente que a atravessa é

dI= Kdy: for a infinitesimal with of the conductor the current the cross it is dI= Kdy

$$\vec{H} = \frac{I}{4\pi}\oint \frac{d\vec{l}}{R^2} \times \vec{a}_R \qquad d\vec{H} = \frac{Kdy}{4\pi}\oint \frac{d\vec{l}}{R^2} \times \vec{a}_R$$

supondo um qualquer ponto, por exemplo (0, 0, Z) tem-se: assuming some point, for example (0, 0, Z)

$$d\vec{H} = \frac{Kdy}{4\pi}\lim_{L\to\infty}\int_{-L}^{L}\frac{d x\,\vec{a}_x}{x^2+y^2+Z^2}\times\frac{0\vec{a}_x+0\vec{a}_y+Z\vec{a}_z-x\vec{a}_x-y\vec{a}_y-0\vec{a}_z}{\sqrt{x^2+y^2+Z^2}} \qquad d\vec{H} = \frac{Kdy}{4\pi}\lim_{L\to\infty}\int_{-L}^{L}\frac{-Z\vec{a}_y-y\vec{a}_z}{\left(x^2+y^2+Z^2\right)^{3/2}}d x$$

$$d\vec{H} = \frac{Kdy}{4\pi}\lim_{L\to\infty}\int_{-L}^{L}\frac{-y\vec{a}_z-Z\vec{a}_y}{\left(x^2+y^2+Z^2\right)^{3/2}}d x \qquad d\vec{H} = \frac{Kdy}{4\pi}\left(-y\vec{a}_z-Z\vec{a}_y\right)\lim_{L\to\infty}\int_{-L}^{L}\frac{d x}{\left(x^2+y^2+Z^2\right)^{3/2}}$$

$$d\vec{H} = \frac{Kdy}{4\pi}\frac{-y\vec{a}_z-Z\vec{a}_y}{y^2+Z^2}\lim_{L\to\infty}\left[\frac{x}{\sqrt{x^2+y^2+Z^2}}\right]_{-L}^{L} \qquad d\vec{H} = \frac{Kdy}{4\pi}\frac{-y\vec{a}_z-Z\vec{a}_y}{y^2+Z^2}2\lim_{L\to\infty}\left(\frac{L}{\sqrt{L^2+y^2+Z^2}}\right)$$

$$d\vec{H} = \frac{K dy}{2\pi}\frac{-y\,\vec{a}_z - Z\,\vec{a}_y}{y^2+Z^2}$$

$$\vec{H} = \lim_{L\to\infty}\int_{-L}^{L}\frac{K dy}{2\pi}\frac{-y\,\vec{a}_z - Z\,\vec{a}_y}{y^2+Z^2}$$

$$\vec{H} = \frac{K}{2\pi}\lim_{L\to\infty}\int_{-L}^{L}\frac{-y\,\vec{a}_z - Z\,\vec{a}_y}{y^2+Z^2}dy$$

$$\vec{H} = -\frac{K}{2\pi}\lim_{L\to\infty}\int_{-L}^{L}\left(\frac{y\,\vec{a}_z}{y^2+Z^2}+\frac{Z\,\vec{a}_y}{y^2+Z^2}\right)dy$$

$$\vec{H} = -\frac{K}{2\pi}\lim_{L\to\infty}\left[\frac{\ln\left(y^2+Z^2\right)}{2}\vec{a}_z + \arctan\left(\frac{y}{Z}\right)\vec{a}_y\right]_{-L}^{L}$$

$$\vec{H} = -\frac{K}{2\pi}\lim_{L\to\infty}\left(2\arctan\left(\frac{y}{Z}\right)\right)\vec{a}_y$$

$$\vec{H} = -\frac{K}{\pi}\arctan(\infty)\vec{a}_y$$

$$\vec{H} = -\frac{K}{\pi}\frac{\pi}{2}\vec{a}_y$$

$$\vec{B} = -\mu\frac{K}{2}\vec{a}_y$$

Resposta Answer

$$\vec{H} = -\frac{K}{2}\vec{a}_y \qquad \vec{B} = -\mu\frac{K}{2}\vec{a}_y$$

6) Um campo magnético dado pela expressão ao lado propaga-se no vazio. A magnetic field given by the expression on the left side propagates in a vacuum.

$$\vec{H} = \frac{K}{r}\cos(\phi)\vec{a}_r$$

$$\begin{cases} -1 < z < 1 \\ -\pi/4 < \phi < \pi/4 \end{cases}$$

a) Encontre a expressão da densidade de fluxo magnético. Find the expression of the magnetic flux density.

$$\vec{H} = \frac{K}{r}\cos(\phi)\vec{a}_r \qquad \vec{B} = \mu_o\mu_r\,\vec{H}$$

Resolução Resolution

$$\vec{B} = \mu_o\mu_r\frac{K}{r}\cos(\phi)\vec{a}_r$$

Resposta Answer

$$\vec{B} = \mu_o\mu_r\frac{K}{r}\cos(\phi)\vec{a}_r$$

b) Encontre o fluxo magnético que atravessa a superfície indicada. Find the magnetic flux through the area indicated at left.

$$\vec{B} = \mu_o \mu_r \frac{K}{r} \cos(\phi) \vec{a}_r \qquad \begin{cases} -1 < z < 1 \\ -\pi/4 < \phi < \pi/4 \end{cases} \qquad \int_S \vec{B} \cdot d\vec{S} = \Phi$$

Resolução Resolution

$$\int_{-\pi/4}^{\pi/4} \int_{-1}^{1} \mu_o \mu_r \frac{K}{r} \cos(\phi) \vec{a}_r \cdot r dz d\phi \vec{a}_r = \Phi \qquad \mu_o \mu_r K \int_{-\pi/4}^{\pi/4} \cos(\phi) d\phi \int_{-1}^{1} dz = \Phi$$

$$\mu_o \mu_r K \int_{-\pi/4}^{\pi/4} \cos(\phi) d\phi 2 = \Phi \qquad 2 K \mu_o \mu_r [\sin(\phi)]_{-\pi/4}^{\pi/4} = \Phi \qquad 2\sqrt{2} K \mu_o \mu_r = \Phi$$

Resposta Answer

$$2\sqrt{2} K \mu_o \mu_r = \Phi$$

7) Considere uma película que se encontra no plano **xy** na qual circula uma corrente com densidade **K** segundo x**.** Encontre a expressão do potencial magnético, **A**. Consider a film that lies in the **xy** plane in which is circulating a current with density **K** oriented along the positive **x**. Find the expression of the magnetic potential **A**.

$$\vec{B} = \nabla \times \vec{A} \qquad \vec{B} = -\mu \frac{K}{2} \vec{a}_y \qquad \nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \vec{a}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \vec{a}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \vec{a}_z$$

Resolução Resolution

$$-\mu \frac{K}{2} \vec{a}_y = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \vec{a}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \vec{a}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \vec{a}_z$$

$$\begin{aligned} 0 &= \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \vec{a}_x \\ -\mu \frac{K}{2} \vec{a}_y &= \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \vec{a}_y \\ 0 &= \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \vec{a}_z \end{aligned} \qquad -\mu \frac{K}{2} \vec{a}_y = \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \vec{a}_y \qquad -\mu \frac{K}{2} = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}$$

como A deve ser independente de x e de y: because A must be independent of x and y:

$$-\mu \frac{K}{2} = \frac{\partial A_x}{\partial z} \qquad \partial A_x = -\mu \frac{K}{2} \partial z \qquad A_x = -\mu \frac{K}{2} (z - z_0)$$

Resposta Answer

$$A_x = -\mu \frac{K}{2} (z - z_0)$$